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# The Thermal Expansion of Pure Metals: Copper, Gold, Aluminum, Nickel, and Iron

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Extremely accurate determinations of the linear thermal expansions have been made interferometrically from −196°C to temperatures about +400°C for Al and +700°C for Fe, Ni, Cu and Au. The relationship between true thermal coefficient of expansion and temperature conforms very well to the Grueneisen-Debye theory when values are chosen for the Debye characteristic temperatures which turn out to agree well with those chosen to achieve agreement with the Debye theory of specific heats. Our values for these characteristic temperatures are: 410°K for Ni, 420°K for Fe, 400°K for Al, 325°K for Cu, and 190°K for Au. The magnetic Curie temperature for Ni is found to be 352°C. In plotting true coefficient of thermal expansion *versus* temperature Simon and Bergmann found a horizontal plateau at about 175° to 235°K for Ni and Fe; but we do not confirm this.

In view of the discrepancies in the recorded values of true coefficients of thermal expansion for pure metals over a rather wide range of temperatures, we thought it worth while to reinvestigate a series of very pure metals from the temperature of liquid nitrogen up to about 700°C, by means of an interferometer dilatometer with photographic recording recently developed in these Laboratories. Another important purpose was to test the validity of the Grueneisen theory[1] of thermal expansion over a similarly wide temperature range.

The Grueneisen theory presents the following expression for the true linear coefficient of thermal expansion as a function of the specific heat and two constants, $Q_0$ and $\frac{1}{6}(m+n+3)$:

$$\beta = C_v \Big/ 3Q_0\left[1-\frac{m+n+3}{6}\frac{E}{Q_0}\right]^2, \tag{1}$$

where $\beta$ is the true linear coefficient of thermal expansion, $C_v$ the specific heat at constant volume; $E=\int_0^T C_v dT$; $m$ and $n$ are the exponents in the attractive and repulsive terms, respectively, in the Mie equation relating the potential energy to the distance between vibrating atoms in a monatomic solid. The constant $Q_0$ is defined in terms of specific heat and coefficient of thermal expansion, or in terms of atomic volume, Grueneisen's constant $\gamma$, and compressibility $K_0$, as follows:

$$Q_0=\frac{(C_P)}{[(1/V)(\partial V/\partial T)]}=\frac{V_0}{\gamma K_0}. \tag{2}$$

The constants necessary to evaluate $\beta$ as a function of temperature are $Q_0$, $m$, $n$, and the Debye constant $\theta$ which determines $C_v$ and $E$; one can, in principle, get their values from measurements of specific heats and compressibility. The constant $\gamma$, commonly known as "Grueneisen's constant," can be obtained either from the Grueneisen relation

$$\gamma=\frac{\text{(thermal expansion) (volume)}}{\text{(specific heat at const. vol.) (compressibility)}} \tag{3}$$

or, as Slater[2] has recently shown, from the constants used in Bridgman's expression relating change in volume to pressure. Slater has shown that values of $\gamma$ thus obtained are in good agreement with those obtained from the Grueneisen relationship. Grueneisen further showed that $\gamma$ is related to the exponent $n$ in the repulsion term in the following manner

$$\gamma=\tfrac{1}{6}(n+2). \tag{4}$$

In the applications of Eq. (1) it has been customary to assume the exponent $m$ of the attractive term to be equal to 3.

The characteristic Debye temperatures $\theta$ used to obtain $C_v$ and $E$ in Eq. (1) are taken from experimental specific heat values.

In actual practice one must fit the constants

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[1] E. Grueneisen, *Handbuch der Physik* (1926), Vol. 10, p. 1.

[2] J. C. Slater, Phys. Rev. **57**, 744 (1940).

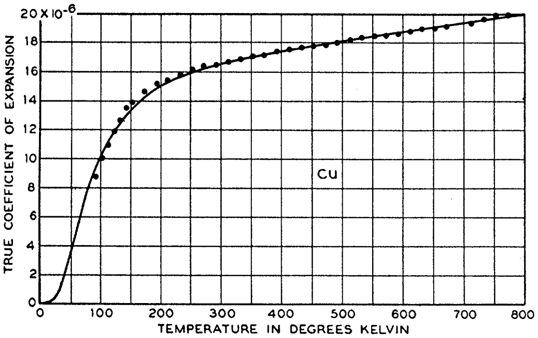

FIG. 1. True coefficient of thermal expansion *vs.* temperature for Cu. The dots are experimentally derived coefficients. The solid curve is the Grueneisen plot with $\theta=325$, $\frac{1}{6}(m+n+3)=2.8$, $Q_0=120\times10^3$ cal.

$\frac{1}{6}(m+n+3)$ and $Q_0$ to at least one point of the experimental data, and in some cases two points, in order to obtain a good fit between the Grueneisen curve and the experimental points. In some cases it was necessary to make slight adjustments in the value of $\theta$ in order to obtain a satisfactory fit. In the captions of the figures given below we give the values of $\theta$, $Q_0$, and $\frac{1}{6}(m+n+3)$ used in order to obtain the agreement between the experimental data and the Grueneisen curve of the respective figures.

## METHOD OF MEASUREMENTS

The measurements were made with an interferometric dilatometer previously described by us,[3] involving photographic recording; it permits slow rates of heating and cooling, so that sufficient time can be allowed for the specimens to come to thermal equilibrium with the refraction thermometer. The specimens were about 6 mm in length. All specimens were sealed in evacuated Pyrex glass tubing and subjected to a prolonged annealing to remove the effects of cold working. The low temperature portion of each test was made at a cooling rate of 20°C per hour from room temperature, with system containing the gas at a pressure of 3 mm of Hg. The fringe systems were photographed at intervals of about 30 seconds, thereby yielding a quasi-continuous record of both expansion and thermometer fringes. The complete unison of motion of the two sets of fringes assured thermal equilibrium of the specimens and refraction thermometer. The temperature was held constant at the lowest temperature obtainable for a period of two hours, in order to obtain a further check on assuring thermal equilibrium. In the temperature region above room temperature, the gas at a pressure of 3 mm of Hg was used in order to facilitate the establishment of thermal equilibrium up to +150°C at a heating rate of 15°C per hour at which point the system was held at a constant temperature while it was evacuated to a pressure of $10^{-5}$ mm of Hg. At higher temperatures the heating rate was increased to 20°C per hour.

TABLE I. *Thermal expansion of copper.*

| $T$ °C | $\Delta l/l_0\times10^4$ | $T$ °C | $\Delta l/l_0\times10^4$ | $T$ °C | $\Delta l/l_0\times10^4$ |
|---|---|---|---|---|---|
| −185.5 | −25.956 | −8.0 | −1.285 | 173.4 | 29.653 |
| −179.0 | −25.443 | −4.5 | − .7710 | 178.0 | 30.607 |
| −173.0 | −24.903 | −1.5 | − .2570 | 183.1 | 31.560 |
| −168.5 | −24.415 | 0 | 0 | 189.0 | 32.514 |
| −164.5 | −23.926 | 1.5 | .2570 | 193.8 | 33.468 |
| −159.5 | −23.387 | 4.2 | .7710 | 199.8 | 34.422 |
| −154.0 | −22.924 | 7.5 | 1.285 | 204.0 | 35.376 |
| −152.0 | −22.410 | 11.0 | 1.799 | 209.8 | 36.330 |
| −147.0 | −21.840 | 13.4 | 2.313 | 215.4 | 37.280 |
| −142.5 | −21.382 | 16.0 | 2.827 | 220.8 | 38.238 |
| −137.5 | −20.817 | 19.2 | 3.341 | 226.2 | 39.192 |
| −134.0 | −20.303 | 22.1 | 3.855 | 232.6 | 40.146 |
| −131.2 | −19.789 | 25.0 | 4.326 | 243.5 | 42.054 |
| −123.0 | −18.761 | 28.5 | 4.807 | 249.0 | 43.007 |
| −119.2 | −18.247 | 31.7 | 5.288 | 259.0 | 44.915 |
| −116.0 | −17.810 | 37.0 | 6.249 | 263.2 | 45.869 |
| −112.0 | −17.219 | 42.3 | 7.211 | 268.1 | 46.823 |
| −108.0 | −16.705 | 47.1 | 8.172 | 273.5 | 47.777 |
| −104.5 | −16.216 | 54.0 | 9.133 | 279.0 | 48.732 |
| −100.5 | −15.677 | 59.6 | 10.095 | 283.8 | 49.685 |
| − 97.2 | −15.163 | 65.5 | 11.056 | 288.8 | 50.639 |
| − 93.5 | −14.649 | 71.0 | 12.018 | 293.0 | 51.593 |
| − 90.5 | −14.160 | 77.0 | 12.979 | 298.8 | 52.547 |
| − 86.1 | −13.621 | 83.0 | 13.941 | 303.7 | 53.501 |
| − 83.2 | −13.107 | 89.0 | 14.902 | 311.9 | 54.931 |
| − 80.2 | −12.644 | 92.0 | 15.383 | 314.0 | 55.408 |
| − 77.0 | −12.130 | 94.5 | 15.863 | 318.8 | 56.362 |
| − 73.2 | −11.565 | 100.7 | 17.252 | 323.7 | 57.316 |
| − 70.0 | −11.051 | 104.0 | 17.729 | 328.8 | 58.270 |
| − 66.2 | −10.537 | 107.0 | 18.206 | 338.2 | 60.178 |
| − 62.0 | −10.023 | 108.9 | 18.583 | 343.9 | 61.132 |
| − 60.2 | − 9.535 | 112.0 | 19.159 | 353.2 | 63.040 |
| − 56.6 | − 8.995 | 114.6 | 19.636 | 362.6 | 64.948 |
| − 53.0 | − 8.481 | 117.0 | 20.113 | 372.1 | 66.855 |
| − 49.0 | − 7.967 | 120.5 | 20.590 | 384.0 | 68.763 |
| − 46.5 | − 7.543 | 122.6 | 21.067 | 394.1 | 70.671 |
| − 44.1 | − 6.965 | 126.0 | 21.544 | 403.8 | 72.579 |
| − 39.5 | − 6.425 | 128.8 | 22.021 | 412.8 | 74.487 |
| − 36.1 | − 5.911 | 132.0 | 22.498 | 422.0 | 76.395 |
| − 33.1 | − 5.397 | 134.7 | 22.975 | 432.5 | 78.303 |
| − 29.6 | − 4.883 | 138.0 | 23.452 | 442.0 | 80.210 |
| − 27.0 | − 4.369 | 140.1 | 23.929 | 452.1 | 82.142 |
| − 23.2 | − 3.855 | 146.0 | 24.883 | 462.0 | 84.026 |
| − 20.3 | − 3.341 | 151.4 | 25.837 | 471.2 | 85.934 |
| − 17.3 | − 2.827 | 157.8 | 26.791 | 481.0 | 87.842 |
| − 14.5 | − 2.313 | 162.5 | 27.745 | 489.7 | 89.750 |
| − 11.0 | − 1.799 | 168.0 | 28.699 | 500.1 | 91.657 |

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[3] F. C. Nix and D. MacNair, Rev. Sci. Inst. **12**, 66 (1941).

TABLE II. *Thermal expansion of gold.*

| $T$ °C | $\Delta l/l_0\times 10^4$ | $T$ °C | $\Delta l/l_0\times 10^4$ | $T$ °C | $\Delta l/l_0\times 10^4$ | $T$ °C | $\Delta l/l_0\times 10^4$ | $T$ °C | $\Delta l/l_0\times 10^4$ | $T$ °C | $\Delta l/l_0\times 10^4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −187.0 | −24.481 | −15.0 | −2.134 | 124.8 | 17.549 | 276.1 | 40.028 | 417.0 | 62.477 | 603.0 | 93.525 |
| −185.0 | −24.049 | −11.0 | −1.586 | 129.5 | 18.213 | 279.6 | 40.633 | 420.5 | 63.054 | 606.0 | 94.101 |
| −179.0 | −23.473 | − 6.1 | − .9805 | 133.0 | 18.675 | 283.8 | 41.210 | 424.0 | 63.630 | 609.8 | 94.677 |
| −175.0 | −22.897 | − 2.7 | − .4037 | 137.6 | 19.336 | 287.0 | 41.786 | 428.0 | 64.207 | 613.8 | 95.311 |
| −170.0 | −22.318 | 0 | 0 | 141.0 | 19.883 | 290.8 | 42.334 | 431.6 | 64.783 | 616.5 | 95.888 |
| −164.5 | −21.741 | 2.0 | .1730 | 145.0 | 20.459 | 294.0 | 42.939 | 435.3 | 65.359 | 619.8 | 96.406 |
| −160.0 | −21.165 | 4.10 | .5189 | 149.5 | 21.036 | 297.7 | 43.515 | 439.7 | 65.936 | 623.7 | 97.040 |
| −154.7 | −20.589 | 10.0 | 1.327 | 152.9 | 21.556 | 301.4 | 44.092 | 443.0 | 66.570 | 628.0 | 97.559 |
| −150.0 | −20.013 | 14.0 | 1.903 | 156.5 | 22.132 | 305.1 | 44.668 | 446.0 | 67.088 | 630.0 | 98.135 |
| −145.0 | −19.436 | 18.0 | 2.480 | 161.0 | 22.767 | 308.0 | 45.215 | 449.8 | 67.665 | 633.0 | 98.712 |
| −140.5 | −18.857 | 22.0 | 3.088 | 164.0 | 23.285 | 311.6 | 45.763 | 453.7 | 68.241 | 636.8 | 99.288 |
| −137.0 | −18.310 | 25.0 | 3.490 | 168.1 | 23.890 | 315.6 | 46.368 | 457.0 | 68.817 | 640.5 | 99.951 |
| −131.1 | −17.734 | 25.0 | 3.489 | 172.5 | 24.467 | 318.8 | 46.916 | 460.8 | 69.394 | 644.0 | 100.49 |
| −126.5 | −17.128 | 29.7 | 4.181 | 175.2 | 25.014 | 322.2 | 47.521 | 463.8 | 69.970 | 647.0 | 101.02 |
| −122.5 | −16.552 | 34.0 | 4.758 | 180.0 | 25.590 | 326.0 | 48.068 | 467.6 | 70.546 | 650.5 | 101.59 |
| −117.3 | −15.973 | 38.0 | 5.334 | 183.2 | 26.166 | 330.0 | 48.674 | 471.6 | 71.123 | 654.0 | 102.17 |
| −112.7 | −15.397 | 42.6 | 5.911 | 188.2 | 26.772 | 333.9 | 49.221 | 475.1 | 71.699 | 657.5 | 102.75 |
| −108.0 | −14.821 | 45.8 | 6.488 | 191.0 | 27.318 | 337.4 | 49.798 | 478.9 | 72.276 | 661.0 | 103.32 |
| −104.0 | −14.244 | 50.1 | 7.093 | 195.6 | 27.924 | 341.0 | 50.403 | 489.0 | 74.005 | 663.0 | 103.90 |
| − 99.5 | −13.668 | 54.0 | 7.641 | 199.0 | 28.471 | 344.8 | 50.950 | 492.0 | 74.581 | 668.0 | 104.48 |
| − 95.0 | −13.092 | 58.5 | 8.247 | 203.0 | 29.047 | 347.8 | 51.527 | 506.2 | 76.829 | 672.0 | 105.11 |
| − 90.5 | −12.516 | 62.0 | 8.795 | 206.8 | 29.653 | 351.3 | 52.103 | 510.5 | 77.578 | 675.0 | 105.63 |
| − 86.2 | −11.939 | 66.0 | 9.371 | 211.0 | 30.229 | 354.8 | 52.679 | 514.8 | 78.183 | 677.0 | 106.20 |
| − 82.0 | −11.360 | 70.0 | 9.948 | 215.0 | 30.864 | 358.2 | 53.256 | 517.3 | 78.673 | 681.0 | 106.78 |
| − 78.5 | −10.784 | 74.2 | 10.525 | 218.2 | 31.355 | 362.5 | 53.832 | 529.8 | 80.748 | 684.8 | 107.36 |
| − 73.8 | −10.208 | 78.8 | 11.101 | 222.4 | 31.931 | 366.0 | 54.408 | 557.0 | 85.744 | 688.0 | 107.93 |
| − 69.0 | − 9.629 | 83.0 | 11.707 | 226.3 | 32.508 | 368.8 | 54.985 | 559.0 | 86.032 | 691.0 | 108.51 |
| − 65.0 | − 9.055 | 87.0 | 12.255 | 230.8 | 33.140 | 373.0 | 55.561 | 568.0 | 87.588 | 694.5 | 109.14 |
| − 60.2 | − 8.479 | 91.0 | 12.832 | 234.6 | 33.716 | 377.0 | 56.137 | 569.1 | 87.819 | 697.6 | 109.66 |
| − 56.5 | − 7.900 | 94.0 | 13.293 | 238.5 | 34.236 | 380.3 | 56.714 | 572.7 | 88.337 | 701.0 | 110.30 |
| − 52.0 | − 7.324 |  | 12.832 | 243.2 | 34.868 | 384.4 | 57.290 | 575.0 | 88.798 | 705.0 | 110.90 |
| − 48.2 | − 6.747 |  | 12.255 | 246.2 | 35.389 | 387.1 | 57.867 | 576.0 | 88.914 | 708.2 | 111.39 |
| − 44.6 | − 6.171 |  | 11.678 | 249.8 | 35.965 | 391.2 | 58.443 | 579.0 | 89.490 | 711.0 | 111.97 |
| − 40.0 | − 5.595 | 100.0 | 14.12 | 254.0 | 36.541 | 395.1 | 59.019 | 583.1 | 90.153 | 714.0 | 112.54 |
| − 36.0 | − 5.045 | 104.2 | 14.638 | 257.6 | 37.118 | 398.8 | 59.596 | 586.0 | 90.700 | 717.0 | 113.18 |
| − 31.2 | − 4.439 | 108.5 | 15.215 | 261.3 | 37.694 | 403.1 | 60.172 | 589.0 | 91.219 | 720.5 | 113.70 |
| − 27.0 | − 3.863 | 112.3 | 15.791 | 264.9 | 38.270 | 405.9 | 60.748 | 592.3 | 91.795 | 723.8 | 114.27 |
| − 23.0 | − 3.287 | 117.0 | 16.370 | 268.8 | 38.847 | 409.9 | 61.325 | 596.0 | 92.429 | 726.0 | 114.85 |
| − 19.0 | − 2.711 | 121.0 | 16.946 | 272.7 | 39.423 | 413.0 | 61.901 | 599.0 | 92.948 | 729.8 | 115.43 |

## EXPERIMENTAL RESULTS

### Copper

The copper used in these experiments was taken from rods produced by Adam Hilger for use as spectroscopic standards which, according to their analysis, possess a copper content of 99.979 percent and oxygen of 0.02 percent. We give in Table I the data obtained, the first column containing the temperature in degrees centigrade and the second column the thermal expansivity, $(\Delta l/l_0)\times 10^4$, where $l_0$ is the length of specimens at 0°C and $\Delta l$ is the change in length. In Fig. 1 is plotted the true coefficient of thermal expansion, $(1/l_0)(dl/dT)$, as a function of temperature in degrees Kelvin; the curve is graphically obtained from the plot of data of Table I. The dots represent the experimental points; the solid curve is the Grueneisen curve obtained with a characteristic Debye temperature of 325, 2.8 for $\frac{1}{6}(m+n+3)$ and $120\times 10^3$ cal. for $Q_0$.

The temperature region from room temperature to 800°K has been previously studied by Eucken and Dannoehl;[4] Rosenbohm;[5] Henning;[6] and Uffelman.[7] The low temperature region has been previously investigated by Adenstedt;[8] Buffington and Latimer;[9] Henning;[6] Dorsey;[10] Keesom, van Agt and Jansen;[11] Borelius and

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[4] A. Eucken and W. Dannoehl, Zeits. f. Elektrochemie **40**, 814 (1934).

[5] E. Rosenbohm, Physica **5**, 385 (1938).

[6] F. Henning, Ann. d. Physik **22**, 631 (1907).

[7] F. L. Uffelman, Phil. Mag. **10**, 633 (1930).

[8] H. Adenstedt, Ann. d. Physik **26**, 69 (1936).

[9] R. M. Buffington and W. M. Latimer, J. Am. Chem. Soc. **48**, 2305 (1926).

[10] H. G. Dorsey, Phys. Rev. **25**, 88 (1907).

[11] W. H. Keesom, F. P.G.A.J. van Agt, and A. F. J. Jansen Proc. k. Acad. Amst. **29**, 786 (1926).

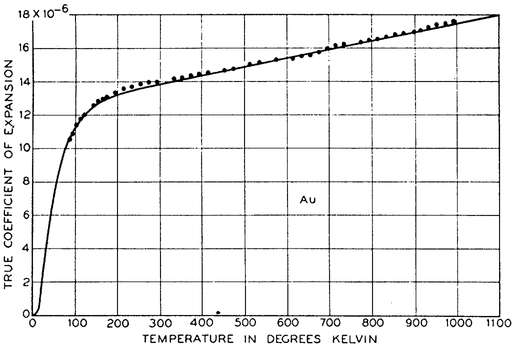

FIG. 2. True coefficient of thermal expansion *vs.* temperature for Au. The dots are experimentally derived coefficients. The solid curve is the Grueneisen plot with $\theta=190$, $\frac{1}{6}(m+n+3)=3.4$, $Q_0=148.8\times10^3$ cal.

Johansson;[12] Simon and Bergmann;[13] and Aoyama and Ito.[14] We are in quite good agreement with the work of Adenstedt; Buffington and Latimer; and Dorsey. The values reported by Simon and Bergmann are much too high near room temperature and too low near 95°K. We are in excellent agreement with the values of $Q_0$ and $\frac{1}{6}(m+n+3)$ obtained by Adenstedt, but we obtain a value of 325 for $\theta$ somewhat higher than Adenstedt's value of 315.

## Gold

The gold used in these expansion experiments was obtained from Handy and Harmon as a spectroscopic standard, containing 99.99+ percent Au, the impurities being traces of Ag, Cu, and Pd. The data are presented in Table **II** with the temperatures given in degrees centigrade and expansivity as $(\Delta l/l_0)\times10^4$. The true coefficient of thermal expansions, obtained graphically from a plot of the data given in Table **II**, is given by the dots as a function of temperature in degrees Kelvin in Fig. 2. The solid line is the Grueneisen curve obtained by using a characteristic Debye temperature of 190, a value of 3.4 for $\frac{1}{6}(m+n+3)$, and $148.8\times10^3$ cal for $Q_0$.

Our results are in good agreement with recent studies of Austin[15] and of Esser and Eusterbrock,[16] in the region from room temperature to 1000°K. The low temperature region has been previously studied by Ebert;[17] Dorsey;[10, 18] and Grueneisen.[19]

## Aluminum

The specimens were cut from a block of high purity aluminum containing 99.997 percent Al.

TABLE III. *Thermal expansion of aluminum.*

| $T$ °C | $\Delta l/l_0\times10^4$ | $T$ °C | $\Delta l/l_0\times10^4$ | $T$ °C | $\Delta l/l_0\times10^4$ |
|---|---|---|---|---|---|
| −191.0 | −34.358 | 43.0 | 10.273 | 233.2 | 57.073 |
| −184.5 | −33.891 | 47.5 | 11.209 | 237.1 | 58.009 |
| −176.5 | −32.932 | 50.7 | 12.145 | 240.2 | 58.945 |
| −169.5 | −31.996 | 55.0 | 13.104 | 244.1 | 59.881 |
| −161.5 | −31.061 | 59.0 | 14.017 | 247.9 | 60.817 |
| −156.0 | −30.125 | 62.8 | 14.953 | 251.0 | 61.753 |
| −148.0 | −29.189 | 67.0 | 15.889 | 254.7 | 62.889 |
| −142.7 | −28.277 | 70.3 | 16.825 | 258.0 | 63.625 |
| −137.0 | −27.342 | 75.9 | 17.761 | 261.1 | 64.561 |
| −130.5 | −26.406 | 79.0 | 18.697 | 264.6 | 65.497 |
| −126.0 | −25.517 | 82.8 | 19.633 | 268.0 | 66.433 |
| −120.0 | −24.512 | 87.0 | 20.615 | 271.0 | 67.369 |
| −115.0 | −23.576 | 91.0 | 21.505 | 275.0 | 68.305 |
| −109.0 | −22.664 | 95.0 | 22.441 | 277.9 | 69.241 |
| −104.0 | −21.775 | 97.6 | 23.377 | 281.0 | 70.177 |
| − 99.5 | −20.816 | 102.8 | 24.359 | 284.5 | 71.113 |
| − 93.6 | −19.834 | 106.4 | 25.272 | 287.5 | 72.049 |
| − 89.0 | −18.898 | 110.5 | 26.231 | 291.0 | 72.985 |
| − 84.0 | −17.963 | 114.0 | 27.121 | 294.0 | 73.921 |
| − 79.5 | −17.027 | 117.8 | 28.057 | 297.1 | 74.857 |
| − 75.3 | −16.092 | 122.5 | 29.086 | 300.6 | 75.793 |
| − 70.5 | −15.156 | 126.0 | 29.929 | 303.7 | 76.729 |
| − 66.0 | −14.225 | 130.0 | 30.888 | 307.0 | 77.665 |
| − 61.3 | −13.285 | 134.4 | 31.801 | 310.0 | 78.601 |
| − 56.5 | −12.349 | 138.0 | 32.760 | 313.0 | 79.537 |
| − 52.5 | −11.413 | 142.0 | 33.673 | 316.0 | 80.473 |
| − 48.0 | −10.478 | 145.1 | 34.632 | 319.7 | 81.409 |
| − 43.5 | − 9.543 | 149.0 | 35.545 | 322.8 | 82.345 |
| − 37.5 | − 8.607 | 153.0 | 36.481 | 326.1 | 83.281 |
| − 34.6 | − 7.672 | 157.0 | 37.417 | 329.2 | 84.217 |
| − 30.1 | − 6.736 | 160.0 | 38.353 | 333.0 | 85.153 |
| − 26.0 | − 5.800 | 162.5 | 38.938 | 336.0 | 86.089 |
| − 22.0 | − 4.865 | 163.0 | 39.289 | 339.0 | 87.025 |
| − 17.0 | − 3.929 | 167.2 | 40.225 | 341.9 | 87.961 |
| − 13.2 | − 2.994 | 171.0 | 41.161 | 345.0 | 88.897 |
| − 9.0 | − 2.058 | 175.0 | 42.097 | 348.0 | 89.833 |
| − 5.0 | − 1.123 | 178.6 | 43.033 | 351.0 | 90.815 |
| 0 | 0 | 182.0 | 43.969 | 354.1 | 91.705 |
| 3.5 | .7484 | 185.9 | 44.905 | 357.0 | 92.641 |
| 7.6 | 1.684 | 189.9 | 45.841 | 360.0 | 93.577 |
| 11.0 | 2.620 | 193.0 | 46.777 | 363.0 | 94.513 |
| 15.1 | 3.555 | 196.7 | 47.713 | 366.1 | 95.449 |
| 19.2 | 4.491 | 200.8 | 48.649 | 369.4 | 96.385 |
| 21.0 | 4.958 | 204.0 | 49.585 | 372.8 | 97.321 |
| 23.0 | 5.426 | 208.0 | 50.521 | 375.5 | 98.280 |
| 25.0 | 5.894 | 211.5 | 51.457 | 379.0 | 99.193 |
| 28.0 | 6.646 | 215.0 | 52.393 | 381.6 | 100.13 |
| 29.0 | 6.997 | 218.9 | 53.329 | 384.7 | 101.06 |
| 31.0 | 7.465 | 222.5 | 54.265 | 387.5 | 102.00 |
| 35.0 | 8.401 | 226.0 | 55.201 | 391.0 | 102.96 |
| 39.0 | 9.337 | 229.7 | 56.137 | 393.8 | 103.87 |

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The heating rate from room temperature to 500°C was 30° per hour. The cooling rate from room temperature to $-191$°C was 20°C per hour. Helium gas at a pressure of 3 mm of Hg was used in low temperature test, and also in the high temperature test up to $+170$°C, at which point the system was evacuated to a pressure of $10^{-5}$ mm of Hg for the remainder of the high temperature test. Table III contains the data tabulated as before. There is a small amount of scattering from 160 to 300°C caused by the lack of thermal equilibrium between specimens and refraction thermometer on evacuation of the gas. A plot of the true linear coefficient of thermal expansion *versus* temperature in degrees Kelvin is given in Fig. 3. The dots represent the data with the solid line giving the Grueneisen plot. This excellent fit between the Grueneisen equation and the experimental data was obtained using a characteristic Debye temperature of 400, a value of 2.7 for $\frac{1}{6}(m+n+3)$, and a value of $83.6\times10^3$ cal for $Q_0$.

Previous measurements in the temperature region from room temperature to 700°K have been made by Hidnert;[20] Uffelman;[7] Henning;[6] and by Taylor, Willey, Smith and Edwards.[21] The low temperature region from room temperature to $-191$°C has been studied by Buffington and Latimer;[9] Henning;[6] and Ebert.[17] A comparison of our values of mean coefficient of thermal expansion with values reported by Hidnert; and Taylor, Willey, Smith, and Edwards is fairly good. We are also in fairly good agreement in the low temperature region with the results of Buffington and Latimer.

## Nickel

The nickel used for the specimens, obtained from the International Nickel Company, contained 99.90 percent Ni, the impurities consisting of 0.03 percent Fe, 0.0035 percent Mg, 0.0005 percent Cu, 0.05 percent C, $<0.001$ percent Mn, 0.0043 percent Al, 0.011 percent $SiO_2$. Table IV contains the experimental data tabulated as before. The true coefficient of thermal expansion *versus* temperature in degrees Kelvin is given in Fig. 4. The dots again represent the experimentally derived coefficients with the theoretical Grueneisen plot given by the full drawn curve. The Grueneisen curve was obtained by using a characteristic Debye temperature $\theta$, of 410, the value of 4 for $\frac{1}{6}(m+n+3)$, and the value of $151.5\times10^3$ cal for $Q_0$. In the case of nickel, in contrast to the metals Au, Cu, and Al, the behavior of the thermal coefficient of expansion is obscured by the presence of a volume change accompanying the vanishing of ferromagnetism. The peak in the $\beta$ *vs.* $T$ plot occurs at 635°K, which is the Curie point. In view of the excellent agreement between experiment and theory for the cases of Cu, Au, and Al, we feel that the Grueneisen curve of Fig. 4 represents the course of $\beta$ *vs.* $T$ insofar as controlled by thermal vibrations, while the deviation beginning at about 320°K is due to the volume change of magnetic origin. It is also of interest to note that the experimental points do not again coincide with the theoretical curve until a temperature of about 780°K is reached, which is some 145° above the Curie point, the peak in the $\beta$ *vs.* $T$ plot.

A detailed plot of temperature coefficient of electrical resistivity *vs.* temperature of nickel of the same purity, from unpublished data, shows a peak in the identical position on the temperature scale. The temperature coefficient of resistivity does not again become constant at higher temperatures until a temperature of about 800°K,

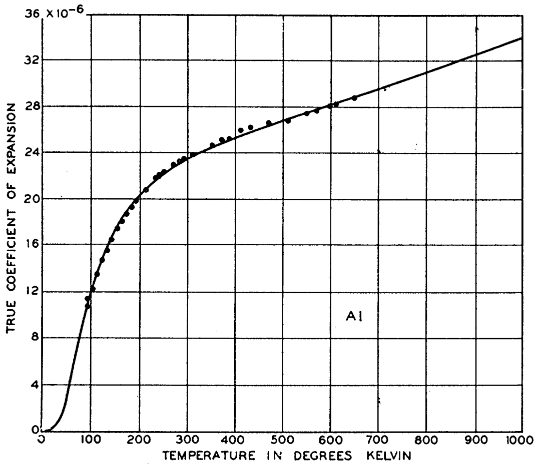

FIG. 3. True coefficient of thermal expansion *vs.* temperature for Al. The dots are experimentally derived coefficients. The solid curve is the Grueneisen plot with $\theta=400$, $\frac{1}{6}(m+n+3)=2.7$, $Q_0=83.6\times10^3$ cal.

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TABLE IV. *Thermal expansion of nickel.*

| $T$ °C | $\Delta l/l_0\times 10^4$ | $T$ °C | $\Delta l/l_0\times 10^4$ | $T$ °C | $\Delta l/l_0\times 10^4$ |
|---|---|---|---|---|---|
| −192.0 | −18.830 | 99.0 | 12.930 | 308.0 | 44.528 |
| −191.0 | −18.579 | 101.0 | 13.243 | 311.6 | 45.018 |
| −184.5 | −18.127 | 104.0 | 13.684 | 314.0 | 45.507 |
| −174.0 | −17.575 | 107.0 | 14.174 | 317.0 | 45.997 |
| −165.5 | −17.072 | 111.1 | 14.663 | 320.2 | 46.487 |
| −158.2 | −16.570 | 115.5 | 15.202 | 322.8 | 47.001 |
| −152.0 | −16.068 | 118.3 | 15.642 | 326.0 | 47.515 |
| −145.0 | −15.616 | 122.6 | 16.156 | 329.0 | 47.980 |
| −139.8 | −15.139 | 125.1 | 16.622 | 331.8 | 48.494 |
| −133.1 | −14.562 | 129.6 | 17.136 | 334.8 | 49.008 |
| −128.0 | −14.060 | 133.8 | 17.650 | 337.1 | 49.473 |
| −122.5 | −13.558 | 137.0 | 18.090 | 340.5 | 49.963 |
| −117.5 | −13.080 | 140.5 | 18.604 | 343.0 | 50.403 |
| −112.0 | −12.553 | 144.0 | 19.143 | 344.7 | 50.648 |
| −107.0 | −12.126 | 147.5 | 19.559 | 346.0 | 50.893 |
| −101.0 | −11.549 | 151.0 | 20.073 | 347.5 | 51.138 |
| − 96.0 | −11.047 | 154.0 | 20.538 | 348.1 | 51.382 |
| − 92.0 | −10.545 | 157.8 | 21.101 | 349.6 | 51.627 |
| − 87.0 | −10.043 | 160.1 | 21.566 | 350.9 | 51.872 |
| − 82.2 | − 9.591 | 163.5 | 22.007 | 352.8 | 52.192 |
| − 78.0 | − 9.089 | 167.0 | 22.546 | 354.0 | 52.362 |
| − 74.0 | − 8.536 | 170.0 | 22.986 | 355.0 | 52.606 |
| − 69.0 | − 8.034 | 173.4 | 23.476 | 357.0 | 52.900 |
| − 64.6 | − 7.582 | 176.2 | 23.990 | 358.1 | 53.096 |
| − 60.0 | − 7.030 | 180.0 | 24.480 | 359.5 | 53.390 |
| − 55.6 | − 6.528 | 182.8 | 24.945 | 360.8 | 53.586 |
| − 51.2 | − 6.026 | 186.0 | 25.483 | 362.0 | 53.830 |
| − 47.0 | − 5.574 | 189.1 | 25.924 | 363.4 | 54.075 |
| − 42.3 | − 5.021 | 192.6 | 26.462 | 364.0 | 54.320 |
| − 38.2 | − 4.569 | 196.2 | 26.927 | 366.2 | 54.565 |
| − 33.3 | − 4.017 | 199.4 | 27.393 | 367.8 | 54.834 |
| − 29.7 | − 3.540 | 203.0 | 27.931 | 369.0 | 55.054 |
| − 25.4 | − 3.013 | 206.0 | 28.372 | 370.6 | 55.299 |
| − 21.1 | − 2.460 | 209.2 | 28.886 | 372.4 | 55.617 |
| − 17.0 | − 2.009 | 213.0 | 29.424 | 373.7 | 55.813 |
| − 13.0 | − 1.506 | 216.0 | 29.914 | 375.0 | 56.034 |
| − 9.0 | − 1.004 | 219.0 | 30.330 | 376.7 | 56.278 |
| − 4.5 | − .502 | 222.7 | 30.869 | 378.0 | 56.572 |
| 0 | 0 | 226.5 | 31.383 | 379.1 | 56.768 |
| 2.0 | .351 | 229.6 | 31.823 | 381.2 | 57.013 |
| 8.1 | 1.004 | 233.3 | 32.362 | 382.5 | 57.282 |
| 12.0 | 1.506 | 236.0 | 32.803 | 385.3 | 57.747 |
| 15.1 | 2.009 | 239.0 | 33.292 | 388.0 | 58.237 |
| 20.1 | 2.511 | 243.0 | 33.806 | 391.6 | 58.726 |
| 23.2 | 3.013 | 246.0 | 34.271 | 394.6 | 59.216 |
| 28.0 | 3.590 | 249.0 | 34.785 | 397.8 | 59.706 |
| 31.2 | 4.017 | 252.1 | 35.299 | 400.8 | 60.195 |
| 36.0 | 4.519 | 255.0 | 36.009 | 404.5 | 60.734 |
| 39.4 | 5.021 | 258.3 | 36.205 | 407.0 | 61.223 |
| 43.0 | 5.523 | 261.3 | 36.768 | 409.0 | 61.664 |
| 47.0 | 6.026 | 264.5 | 37.184 | 413.0 | 62.153 |
| 51.0 | 6.528 | 268.0 | 37.723 | 415.5 | 62.668 |
| 55.0 | 7.030 | 270.3 | 38.164 | 418.5 | 63.133 |
| 58.8 | 7.532 | 273.6 | 38.678 | 422.0 | 63.647 |
| 62.2 | 8.034 | 279.2 | 39.657 | 424.0 | 64.112 |
| 66.0 | 8.536 | 282.3 | 40.146 | 427.1 | 64.601 |
| 69.8 | 9.038 | 285.0 | 40.611 | 430.0 | 65.140 |
| 74.0 | 9.540 | 288.8 | 41.126 | 434.0 | 65.630 |
| 77.7 | 10.043 | 291.2 | 41.591 | 442.7 | 67.049 |
| 81.5 | 10.545 | 294.0 | 42.080 | 454.0 | 69.008 |
| 85.4 | 11.047 | 297.0 | 42.643 | 467.2 | 70.966 |
| 89.1 | 11.549 | 299.0 | 43.108 | 477.5 | 72.924 |
| 93.4 | 12.051 | 303.0 | 43.574 | 486.2 | 74.491 |
| 97.1 | 12.553 | 305.7 | 44.039 | 488.5 | 74.883 |

a behavior very like that of the expansion coefficient. Mott and Potter explain such "tails" extending over a rather wide range by the assumption that at the Curie point the large Weiss domains break up, leaving something of the nature of short range order to vanish gradually as $T$ is further increased.

The Heisenberg theory of ferromagnetism, as pointed out by Fowler and Kapitza[22] and later demonstrated by Powell,[23] can account in a rough quantitative way for the change in volume which ferromagnetic substances undergo as the ferromagnetism vanishes on heating. The first careful experimental attempt to obtain the ferromagnetic change in length was made by Williams[24] on quite pure nickel. In order to obtain the area under the Curie peak Williams used as a base line his own data at temperatures from room temperature to about 300°C and those of Hidnert[25] for temperatures considerably above the Curie point. We feel in view of the excellent agreement obtained between theory and experiment for Cu, Au, and Al that a theoretical Grueneisen curve which fits the experimental points at temperatures around room temperature and below would serve as a much more accurate base line than the experimental curve used by Williams. The increase in volume due to the vanishing of ferromagnetism begins as low as 325°K in Fig. 4 whereas in Fig. 5 of Williams' paper it is not perceptible until about 525°K is attained.

A careful measurement of the area between the Grueneisen and the experimental curves gives the value $3.65\times 10^{-4}$ for the ferromagnetic change in length per unit length, compared with Williams' value of $0.92\times 10^{-4}$ for nickel of comparable purity.

We are in quite good agreement with the previous experimental values of Williams.[24] Other work at temperatures above room temperature includes the studies of Hidnert;[25] Henning;[6] Disch;[26] Uffelman;[7] Eucken and Dannoehl;[4] Rosenbohm;[5] Colby,[27] and x-ray studies at high

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[27] W. F. Colby, Phys. Rev. **30**, 506 (1910).

temperatures by Owen and Yates[28] and also by Jesse.[29] We are in good agreement with recent low temperature work of Adenstedt.[8] Other low temperature work includes that of Henning;[6] Disch;[26] Aoyama and Ito,[14] and also of Simon and Bergmann.[13] We do not confirm the existence of a flat plateau in the $\beta$ *vs.* $T$ plot for the temperature region from 175 to 235°K reported by Simon and Bergmann, nor do we agree with the value of $\beta$ at any temperature.

## Iron

The iron used in this investigation was kindly furnished by Dr. J. G. Thompson of the United States Bureau of Standards. He reports the impurities to be

Cu—$<$0.002 percent
Si—0.001 percent
Be—$<$0.001 percent
Al—Nil
C—0.001 percent
S—0.0023 percent
P—0.0005 percent
$O_2$—0.0003 percent
$N_2$—0.0002 percent
$H_2$—0.0002 percent
Fe—99.992 (by difference)

The iron specimens were heated in He gas at a pressure of 3 mm of Hg up to 100°C at a rate of 10°C per hour, then in a vacuum of $10^{-5}$ mm of Hg up to 800°C. The low temperature test was made in an atmosphere of He at 3 mm of Hg at a cooling rate of 20°C per hour. Table V contains data on the expansion at different temperatures. The dots in Fig. 5 again give the graphically derived true coefficients of thermal expansion as a function of temperature in degrees Kelvin, with the solid curve giving the Grueneisen plot. We note that the fit is much worse than in the case of Ni and we are forced to the conclusion that deviations from the Grueneisen curve at temperatures above 200°K are to be attributed to disturbances arising from the slow disappearance of ferromagnetism with increasing temperature. The Grueneisen curve was obtained with the characteristic Debye temperature of 420, the value of 3.7 for $\frac{1}{6}(m+n+3)$, and the $Q_0$ value of $166.67\times10^3$ cal.

In the temperature region above room temperature we find ourselves in rather good qualitative agreement with the results of Austin and Pierce,[30] and with those of Esser and Eusterbrock.[16] The variations from sample to sample are quite considerable in the region above room temperature where presumably the effects of extremely small amounts of impurities can materially affect the ferromagnetic change in length. In the low temperature region we are in rather good agreement with the results of Adenstedt[8] and somewhat in poorer agreement with the results of Dorsey[10] and of Ebert.[17] Again, as in the case of Ni we fail to find any evidence of the flat plateau extending from 195 to 235°K as reported by Simon and Bergmann. Furthermore we fail to agree with them as to values of the coefficient throughout the region.

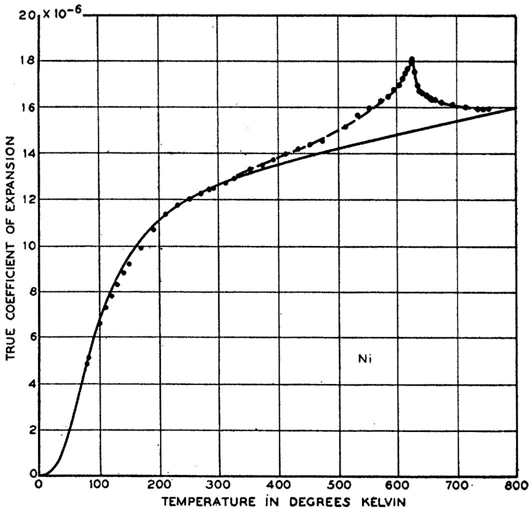

FIG. 4. True coefficient of thermal expansion *vs.* temperature for Ni. The dots are experimentally derived coefficients. The solid curve is the Grueneisen plot with $\theta=410$, $\frac{1}{6}(m+n+3)=4.0$, $Q_0=151.5\times10^3$ cal.

### DISCUSSION OF RESULTS

The agreement between our experimental results and the Grueneisen theory is excellent in the cases of the metals Cu, Au and Al, i.e., in metals free of complications arising from volume changes due to slow disappearance of ferromagnetism with increasing temperature. In the case of Ni the fit is good below 325°K which indicates that the thermal expansion is to be attributed solely to the normal lattice vibrations

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[30] J. B. Austin and R. H. H. Pierce, Jr., Physics **4**, 409 (1933).

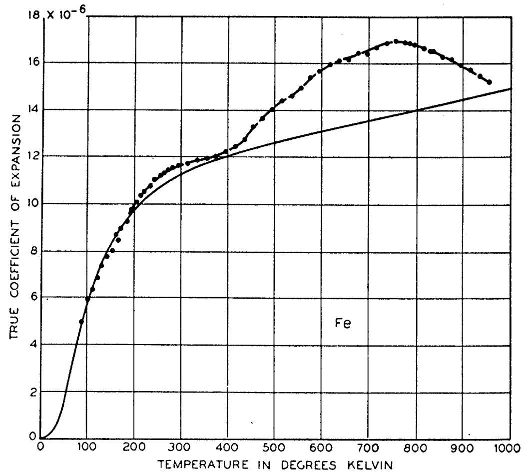

FIG. 5. True coefficient of thermal expansion *vs.* temperature for Fe. The dots are experimentally derived coefficients. The solid curve is the Grueneisen plot with $\theta = 420$, $\frac{1}{6}(m+n+3) = 3.7$, $Q_0 = 166.67 \times 10^3$ cal.

considered in the Grueneisen theory. Above this temperature, where the disappearance of ferromagnetism produces an increase in volume, the total thermal expansion is much greater up to a temperature some 135° above the peak or Curie point. Since such considerations are not involved in the basis of the Grueneisen theory, we should not expect the latter to agree with the experimental results. For the same reasons we should also expect no agreement between theory and experiment for the case of Fe.

The theoretically expected values for $\frac{1}{6}(m+n+3)$,—obtained by assuming $m=3$ along with values of $n$ taken from the Grueneisen constant $\gamma$ with the aid of Eq. (4)—are 2.68, 3.70, 2.83, 2.55, and 2.26 for Cu, Au, Al, Ni, and Fe, respectively. These values are in moderately good agreement with the values used in order to obtain a good fit between the theoretical Grueneisen curve and the data of 2.8, 3.4, 2.7, 3.7 and 4.0, respectively.

It is of interest to note that small amounts of impurities appear to play only a minor role in the thermal expansion of non-ferromagnetic metals, and also a vanishingly small role in the ferromagnetic metals at temperatures sufficiently low for the temperature dependence of the ferromagnetism to play a small part in the thermal expansion. The good agreement obtained between our results and those of Taylor, Willey, Smith, and Edwards[21] on the very pure Al at high temperatures compared with results obtained by Hidnert[20] using Al of a substantially less degree of purity indicate that the small amounts of impurities do not play a very important role. The same can be said for the agreement between our results on Al at low temperatures and those of Buffington and Latimer[9] obtained with rather impure material.

TABLE V. *Thermal expansion of iron.*

| $T$ °C | $\Delta l/l_0 \times 10^4$ | $T$ °C | $\Delta l/l_0 \times 10^4$ | $T$ °C | $\Delta l/l_0 \times 10^4$ |
|---|---|---|---|---|---|
| −181.5 | −16.607 | 81.0 | 9.710 | 385.1 | 53.830 |
| −176.0 | −16.109 | 85.2 | 10.210 | 392.0 | 54.806 |
| −166.0 | −15.241 | 90.0 | 10.711 | 398.5 | 55.757 |
| −159.0 | −14.740 | 93.0 | 11.211 | 405.2 | 56.808 |
| −148.0 | −14.240 | 97.1 | 11.712 | 410.7 | 57.759 |
| −142.0 | −13.739 | 102.9 | 12.262 | 417.1 | 58.760 |
| −134.0 | −13.289 | 106.2 | 12.713 | 423.2 | 59.761 |
| −127.0 | −12.738 | 111.6 | 13.213 | 430.0 | 60.837 |
| −121.0 | −12.237 | 115.2 | 13.714 | 435.3 | 61.788 |
| −114.5 | −11.737 | 119.9 | 14.265 | 441.3 | 62.764 |
| −109.5 | −11.236 | 123.0 | 14.715 | 447.0 | 63.765 |
| −103.5 | −10.786 | 127.9 | 15.241 | 453.0 | 64.766 |
| − 98.0 | −10.235 | 132.0 | 15.716 | 459.0 | 65.767 |
| − 92.5 | − 9.735 | 136.0 | 16.267 | 465.0 | 66.768 |
| − 87.0 | − 9.234 | 141.0 | 16.767 | 471.0 | 67.769 |
| − 81.2 | − 8.734 | 144.0 | 17.218 | 477.0 | 68.770 |
| − 77.0 | − 8.233 | 147.2 | 17.768 | 483.5 | 69.771 |
| − 73.0 | − 7.783 | 151.0 | 18.219 | 488.8 | 70.772 |
| − 67.1 | − 7.232 | 155.1 | 18.769 | 495.0 | 71.773 |
| − 62.8 | − 6.732 | 161.6 | 19.720 | 501.0 | 72.774 |
| − 57.8 | − 6.231 | 169.2 | 20.721 | 507.0 | 73.775 |
| − 53.3 | − 5.806 | 176.5 | 21.797 | 513.0 | 74.776 |
| − 48.0 | − 5.230 | 183.0 | 22.723 | 519.0 | 75.777 |
| − 42.2 | − 4.730 | 190.0 | 23.724 | 525.0 | 76.778 |
| − 38.0 | − 4.229 | 197.0 | 24.800 | 531.5 | 77.779 |
| − 33.2 | − 3.729 | 205.0 | 25.726 | 537.0 | 78.780 |
| − 29.0 | − 3.228 | 212.0 | 26.802 | 543.0 | 79.781 |
| − 25.0 | − 2.728 | 218.1 | 27.728 | 549.0 | 80.882 |
| − 20.5 | − 2.227 | 226.1 | 28.804 | 555.2 | 81.883 |
| − 16.0 | − 1.727 | 232.3 | 29.730 | 561.0 | 82.784 |
| − 11.0 | − 1.226 | 239.6 | 30.731 | 567.0 | 83.785 |
| − 6.7 | − .7257 | 246.2 | 31.732 | 573.0 | 84.786 |
| − 2.7 | − .2252 | 252.9 | 32.733 | 579.0 | 85.787 |
| 0 | 0 | 260.0 | 33.734 | 582.8 | 86.438 |
| 2.5 | .3504 | 266.0 | 34.735 | 584.2 | 86.788 |
| 5.8 | .7758 | 272.7 | 35.736 | 590.5 | 87.789 |
| 10.5 | 1.276 | 279.0 | 36.737 | 597.9 | 88.841 |
| 14.0 | 1.777 | 285.2 | 37.738 | 602.8 | 89.842 |
| 18.0 | 2.277 | 292.1 | 38.739 | 609.0 | 90.793 |
| 22.0 | 2.778 | 298.1 | 39.740 | 615.8 | 91.844 |
| 28.0 | 3.304 | 304.5 | 40.742 | 622.1 | 92.795 |
| 32.5 | 3.454 | 310.8 | 41.793 | 627.5 | 93.796 |
| 36.0 | 4.204 | 317.0 | 42.794 | 635.2 | 94.797 |
| 40.3 | 4.705 | 323.8 | 43.770 | 641.2 | 95.798 |
| 45.1 | 5.205 | 329.8 | 44.796 | 648.5 | 96.899 |
| 48.8 | 5.706 | 336.0 | 45.822 | 655.0 | 97.900 |
| 53.0 | 6.206 | 342.0 | 46.748 | 660.1 | 98.801 |
| 57.0 | 6.707 | 348.0 | 47.774 | 667.0 | 99.802 |
| 61.5 | 7.207 | 354.1 | 48.800 | 674.3 | 100.90 |
| 65.0 | 7.708 | 360.5 | 49.801 | 678.0 | 101.30 |
| 68.9 | 8.208 | 362.2 | 50.752 | 681.0 | 101.91 |
| 73.0 | 8.709 | 373.0 | 51.753 | 684.8 | 102.90 |
| 77.8 | 9.209 | 379.8 | 52.804 | | |

In ferromagnetic metals such as Ni and Fe, on the other hand, the presence of small amounts of impurities can play a dominating role in determining the total thermal expansion for certain temperature regions. This is clearly indicated for the case of Fe by a comparison of our results with those of Austin and Pierce[30] and of Esser and Eusterbrock,[16] as well as results obtained by these workers on irons of slightly different purity. For the temperature region between room temperature and the temperature of liquid nitrogen the role played by small amounts of impurities in Fe is much smaller than in the high temperature region, as evidenced by the rather good agreement between our results and those of Adenstedt[8] and also the older results of Dorsey.[10] Williams[24] demonstrated rather clearly the influence of small amounts of impurities on the thermal expansion for the case of nickel near the Curie temperature.

## ACKNOWLEDGMENT

We wish to thank Dr. L. A. MacColl for many helpful discussions of the Grueneisen theory and Dr. K. K. Darrow for his comments in the preparation of the manuscript.

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# Rigidity Modulus of Beta-Brass Single Crystals

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The rigidity modulus of ten beta-brass single crystals has been measured as a function of crystal orientation and of temperature from 25° to 500°C by the method of the composite piezoelectric oscillator. The reciprocal of the rigidity modulus, $1/G'$, is linearly related to the orientation function. When these data are combined with Rinehart's previous measurements of Young's modulus, the principal elastic parameters are found to be 3.88, $-1.52$ and $0.578\times 10^{-12}$ cm$^2$/dyne, respectively, at room temperature. Curves and tables give them as functions of temperature up to and slightly beyond the critical temperature for order-disorder. At room temperature, the rigidity modulus is a maximum in the [100] direction, $G'_{[100]}=17.3\times 10^{11}$ dyne/cm$^2$, and a minimum in the [111] direction, $G'[111]=1.35\times 10^{11}$ dyne/cm$^2$, and at the critical temperature, $G'[100]=13.5\times 10^{11}$ dyne/cm$^2$ and $G'[111]=1.06\times 10^{11}$ dyne/cm$^2$. The elastic anisotropy as given by $G'[100]/G'[111]$ is 12.8 at room temperature, decreases to 12.3 at 250°C and has a value of 13.0 at the critical temperature. The bending-torsion effect was found to be large and in agreement with theory.

## INTRODUCTION

THE transition from "order" to "disorder" in certain binary alloys has been studied in terms of changes in various physical properties during this transition.[1] The study of the elasticity of single crystals has been particularly interesting, although a complete theoretical treatment has not been given. Siegel[2] has determined all the principal elastic constants (and corresponding elastic parameters) for $Cu_3Au$ single crystals, from room temperature up to and beyond the critical temperature for order-disorder. Both Webb[3] and Rinehart[4] have worked with single crystals of beta-brass. The former's work included a preliminary study of Young's modulus at room temperature and as a function of the orientation of the crystals. A static method was used. Rinehart, using a more accurate dynamic method extended the work on Young's modulus from about $-80°$ to 500°C (about 30° above the critical temperature). The dependence on orientation was very precisely determined. Rinehart's data established, throughout the temperature range, the elastic parameter, $s_{11}$, and a

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